Book Reviews

Stephen A. Vavasis, *Nonlinear Optimization: Complexity Issues*, The International Series of Monographs on Computer Science, Vol. 8, Oxford University Press, New York, 1991, x + 165 pp., 24 cm. Price $39.95.

This is a comprehensive, interesting, and very well-organized book containing carefully selected topics on nonlinear programming and complexity issues. The book has six chapters. The main part consists of Chapters 3–5 which are devoted to complexity issues of quadratic programming.

Chapters 1 and 2 introduce basic materials about optimality conditions and computational complexity classes, including a discussion on computation models for numerical computing. What model is suitable for numerical computing? It is a question which generates many arguments in the literature. In this book, three reasonable criteria are raised: 1. Do results proved with the model correspond to practical experience? 2. Does the model correspond to actual computers? 3. Does the model correspond to the way that algorithm designers think about algorithms?

As an important fact on quadratic programming, it is proven in Chapter 2 that, in general, quadratic programming is NP-hard. Due to this fact the study on special cases of quadratic programming is of interest. In Chapter 3, convex quadratic programming is studied, starting with a strongly polynomial time algorithm for separable quadratic knapsack problems. Since it is an open problem whether or not there exists a strongly polynomial-time algorithm for convex quadratic programming, only a polynomial-time (but not strongly polynomial-time algorithm) interior point algorithm is presented for the general convex quadratic programming problem.

Chapter 4 is devoted to nonconvex quadratic programming. Several special cases are considered in this chapter: nonconvex quadratic optimization problems over the unit simplex and problems with box constraints are proven to be NP-hard, while an efficient algorithm is given for the problem with an ellipsoid constraint.

Chapter 5 studies the problem of finding local minimum points of non-convex quadratic programming and its complexity. It is proven that this problem is still NP-hard. It is worth mentioning that recently P. M. Pardalos proved that the problem of checking the existence of a Kuhn–Tucker point for nonconvex quadratic programming is also NP-hard. However, the complexity of the problem of computing a Kuhn–Tucker point is still not known. This is interesting because the problem is so basic for nonlinear optimization. Although the general problem of computing a local minimum is hard, a strongly polynomial algorithm is presented for computing a local minimum of separable quadratic knapsack problems.

In Chapter 6, the author comes back to the problem of computation models for numerical computing. Among current models, Friedman–Ko's model (based on Turing machines), Smale–Blum's model, and Traub's model (information-based complexity or black-box model), the author chooses the third for further discussions on global optimization. It is shown that the worst-case complexity for global minimization is exponential in the number of variables and the number of digits of accuracy; however, in the case of local minimization, the dependence on the number of digits remains exponential, but the running time is polynomial in the number of variables.

Summarizing from the above, I think that the book provides a valuable source for researchers and students who are interested in complexity issues in nonlinear optimization. I highly recommend this book. It is not only a good reference but also a good textbook in this area.

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DING-ZHU DU

Ding-Zhu Du, *Convergence Theory of Feasible Direction Methods*, Science Press New York, LTD. and Science Press Beijing, China, 1991. iv + 118 pp. ISBN 1-880132-00-1. A volume in Discrete Mathematics and Theoretical Computer Science series.

The convergence of the gradient projection method, an algorithm first proposed by J. B. Rosen in 1960, was a long-standing open problem until recently. Textbooks on nonlinear programming, e.g., Bazaraa and Shetty (1979), would simply state that there is no known convergence proof or a counter example for Rosen's gradient projection method. However, the method itself is intuitively appealing, serves as a foundation of many optimization algorithms in existence today, and always converges to a KKT (stationary) point in practice. It was only in the mid 1980's that a group of Chinese researchers, one of whom is the author of this monograph, finally established the convergence for Rosen's method, a major accomplishment in nonlinear programming.

This monograph summarizes the development and extensions of the convergent proof, some of which have been published either in Chinese or in journals which may not be easily accessible. Since the monograph is also based on a lecture series the author gave at a graduate summer school, it contains some exercises and requires some knowledge of linear programming and mathematical analysis at the level required by most standard textbooks on nonlinear programming. One major difference is the fact that many of the results are new and cannot be found in any other textbooks published prior to 1991. In addition, the monograph focuses on optimization problems with linear constraints, which makes it more suitable as a supplementary text for a senior or graduate level course in optimization. For scientists and researchers, this monograph would be a good source for the new convergence theory which is more general than those based on closed mappings or other standard proving techniques.

To summarize the monograph, Chapter 1 provides an introduction, basic notations, optimality conditions and line search techniques for optimization problems. Chapter 2 introduces two of the three 'Slope' lemmas. The name 'Slope' is from the mountain climbing analogy. Suppose one wants to construct a road to the top of a mountain. One can make the road less steep by lengthening it. As the length of the road approaches infinity, the road becomes flatter and its slope approaches zero. When an algorithm constructs a sequence of solutions $\{x_k\}$, they implicitly describes a road to the top of a mountain. If the sequence is infinite, the length of the road is infinite and the slope of at the tail end of the road must be flat, i.e., zero slope, thereby signifying that there is no *improving* feasible direction. Under suitable conditions, these Slope lemmas lead to the convergence proof for Rosen's method, which is the topic of Chapter 3. Chapter 4 describes variable metric extensions of Rosen's method. Chapters 5 and 6 describe variations of the ϵ-active set strategy and the reduced gradient method, respectively. The convergence proof for all these methods are based on the Slope lemmas. Chapter 7 discusses the consequences of the Slope lemmas with respect to Zangwill's point-to-set mapping and points out why the standard approach (see, e.g., page 387 of Bazaraa and Shetty, 1979) for proving convergence fails when applied to Rosen's method. Finally, Chapter 8 describes the third Slope lemma and its consequences. Also included in this chapter is a brief discussion on Karmarkar's interior point method and nondifferentiable optimization.

Reference

Bazaraa, M. S. and Shetty, C. M., *Nonlinear Programming: Theory and Algorithms*, John Wiley and Sons, New York, New York, 1979.

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